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Can Particle Creation by a Black Hole be Described in Terms of More Familiar Laboratory Processes?

R. M. Nugayev[1]

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Particle creation by a black hole is described in terms of temperature corrections to the Casimir effect. The results of Levin, Polevoy, and Ritov for spectral and total Poynting vector for a fluctuating electromagnetic field in a plane vacuum gap between two arbitrary media with different temperatures in flat spacetime are applied to clarify the situation that exists between the horizon of a nonrotating black hole and spatial infinity. This helps to reveal the mechanism of particle creation. The Hawking radiation is "born" inside the "bell" formed by a potential barrier of a black hole in all the region $[2M, \infty]$. Its blackbody spectrum is due to the interaction of field fluctuations with the surface of the "bell." The particles between the "walls" are virtual ones. They can become real after passing through the $[3M, \infty]$ tail, appearing to an observer at future infinity J^+ as "real" ones. The arguments for and against the present standpoint are discussed.

In previous papers (Nugayev, 1982, 1985; Nugayev and Bashkov, 1979) a program of reducing particle creation by a black hole to quantum field effects in flat space-time was initiated. The program is based on the fact that the gravitational field of a black hole creates an effective potential barrier that is penetrable for high-frequency waves and impenetrable for waves with low frequency. The barrier is so well-localized near $r = 1.5Rg$ ($Rg = -2\ GM/c^2$) that for the study of wave propagation we can consider the regions quite near the horizon and far away from it as "flat." All the scattering takes place in the small region near $r = 1.5Rg$. Consideration of the barrier peak ($r = 1.5Rg$) as a surface of a reflecting sphere permits us to apply to a black hole the results of various Casimir-effect calculations. It appeared (Nugayev and Bashkov, 1979) that the flow of negative Casimir energy should cause the area of the horizon to shrink at a rate consistent with the energy flux observed at future infinity J^+. But the model appeared to be too primitive, providing only qualitative agreement with Hawking's (1975) results.

[1]Kazan State Pedagogical Institute, Kazan, USSR 420021.

Hence, a second stage of the program had to be carried out (Nugayev, 1982). It consisted in the creation of a more sophisticated model capable of demonstrating that the two properties of a black hole—the horizon and the potential barrier—together are necessary and sufficient to compel the hole to produce thermal radiation at a temperature that exactly coincides with the result of Hawking. This was done by reducing the evaporation effect to that of particle creation by (nonuniformly) accelerated mirrors.

But even the second model remained too simple to mimic some important features of the evaporation process, since the vacuum stress-tensor diverged in the reference frame of a freely falling observer as $r \to 2M$. The pathology of the second model is due to the assumption of ideal conductivity, which is obviously not the case for the spherical potential barrier of a nonrotating black hole. So, a third stage of the program was carried out (Nugayev, 1985) and the finite conductivity of the barrier was taken into account. It make it possible to eliminate the pathology and to reveal simultaneously that the blackbody radiation should be "created" in the whole region $[3M, \infty]$.

However, even the third model is able to describe the creation domain only, but not the *mechanism* of black hole evaporation or *why* the radiation at J^+ is the blackbody radiation. The cause of the difficulty is obvious: the third model ignores the thickness of the potential barrier. In our earlier work the barrier was approximated by a *thin* shell. However, Fabbri (1975) demonstrated that there are *two* branches of turning points for a nonrotating black hole potential barrier:

$$r_1 = \frac{2}{\omega}\left[\frac{l(l+1)}{3}\right]^{1/2} \cos\frac{\eta}{3}$$

$$r_2 = \frac{2}{\omega}\left[\frac{l(l+1)}{3}\right]^{1/2} \cos\frac{\eta - 2\pi}{3}$$

where $\eta = \arccos\{-3\omega M[3/l(l+1)]^{1/2}\}$ and arccos denotes the principal value of the inverse trigonometric function, so that $2M \leq r_1 \leq r_2$. For instance, each (ω, l) partial low-frequency wave has *two* turning points:

$$\begin{aligned} r_1 &= 2M\left[1 + \frac{4\omega^2 M^2}{l(l+1)} + O\left(\frac{\omega^4 M^4}{l^4}\right)\right] \\ r_2 &= \frac{[l(l+1)]^{1/2}}{\omega} - M\left[1 + O\left(\frac{\omega M}{l}\right)\right] \end{aligned} \qquad \text{(i)}$$

where $O(x)$ denotes a quantity of order x.

Consequently, for investigating the interaction of virtual particles with the surface of the potential barrier, the latter should be represented by *two* conducting concentric spheres. One of the shells is situated near the horizon, while the other is far from it. Each sphere is made of an ideal conductor. The aim of this paper is to give a description of the *evaporation mechanism* on the basis of this model.

Casimir (1948) demonstrated that the vacuum fluctuations of the electromagnetic field give rise to an attractive force between conducting parallel plates. When one quantizes the field subject to the appropriate boundary conditions on the plates and calculates the vacuum energy with a wavelength cutoff, one finds that as the separation between the plates changes, the vacuum energy per unit area changes by a finite, cutoff-independent amount. Thus, in spite of the formal divergence of vacuum energy, a change in the configuration of the system causes a finite shift in the energy of the vacuum state. If the vacuum energy of the system for infinite separation is set equal to zero, then the energy of the plates for any finite separation is described by the expression

$$\Delta E = -\pi^2 \hbar c A / 720 d^3 \tag{1}$$

Here A denotes the area of each plate and d is the finite separation between them.

Note that the Casimir energy is of *pure vacuum* origin. No real particles are involved, only virtual ones. But the experiments of Derjagin, Sparnaay, van Silfhout, Tabor, and Winterton (see Boyer, 1970, and references cited therein) encourage us to take it seriously.

For the electromagnetic field the stress tensor of the vacuum between the plates was calculated by De Witt (1975):

$$\begin{aligned}\langle T^{\mu\nu}\rangle_{\text{vac}} &= T^{\mu\nu}_{(-)} + T^{\mu\nu}_{(+)} \\ &= \frac{\pi^2 \hbar c^2}{720 d^4} \operatorname{diag}(-1, 1, 1, 3) + \frac{3\Lambda^4 \hbar c^2}{\pi^2} \operatorname{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3})\end{aligned} \tag{2}$$

where Λ is a frequency cutoff that cuts off the high-frequency waves. (The expression for the vacuum stress tensor of a massless scalar field differs from that for the electromagnetic field only by the factor $\frac{1}{2}$.)

The work of Boyer (1968, 1970) offers a method for calculating the vacuum energy of an uncharged sphere made from a physically realizable conductor. Let us approximate a sphere of radius d by two parallel plates of area πd^2 at a distance d apart. With the help of (1) and (2) we obtain

$$\Delta E = -\pi^2 \hbar c / 720 d + 3 \hbar c \Lambda^4 d^3 / \pi \tag{3}$$

where the second part is a correction for the finite conductivity of the plates. The approximation is justified by the exact calculations of Boyer (1968) and Davies (1972) performed independently. Having computed the vacuum energy of a sphere with ideal conductivitiy, they demonstrated that ΔE exactly coincides in magnitude with the cutoff-independent part of (3). Only the sign changes. So, for finite conductivity

$$\Delta E = \pi^3 \hbar c/720d - 3\hbar c\Lambda^4 d^3/\pi \tag{4}$$

All the studies of massless fields in various metallic cavities, initiated by Boyer and reconsidered by several groups, have focused on the integrated total energy. Olaussen and Ravndal (1981) seem to have been among the first to undertake the more detailed and laborious analysis of local densities for spherical cases.

The energy density is given by

$$\langle\Omega|:u(r):\Omega\rangle \simeq -\frac{1}{d^4}\left\{\frac{4}{15\pi^2\delta^3} - \frac{2}{35\pi^2\delta^2} + 0.005\left[1+\left(\frac{r}{d}\right)^2\right]\right\} \tag{5a}$$

where $\delta = [1-(r/d)^2]$.

Near the surface of the sphere where $r \to d$

$$\langle\Omega|:u(r):\Omega\rangle \cong \frac{1}{\pi^2 d^4}\left(\frac{1}{30\varepsilon^3}\right) \tag{5b}$$

where $\varepsilon = 1 - r/d$. So, the (normal ordered) energy density is found to be negative everywhere inside the cavity. In that respect a spherical cavity is not qualitatively different from the parallel plate or box geometries. The unexpected result, first found by Boyer, that the total Casimir effect for a spherical shell actually leads to an increase in energy, only occurs because the energy density outside the shell is positive enough to overcompensate the negative energy inside.

The stresses induced in the Minkowski vacuum by an infinite plane conductor that is uniformly accelerated normal to itself were investigated by Candelas and Deutsch (see Sciama et al., 1981). The solution of the boundary problem was facilitated by the introduction of accelerated (Rindler) coordinates ξ and τ:

$$t = \xi \operatorname{sh} \tau, \qquad x = \xi \operatorname{ch} \tau, \qquad ds^2 = \xi^2\, d\tau^2 + d\xi^2 + dy^2 + dz^2$$

In this system the curves $\xi = \text{const}$, $y = \text{const}$, $z = \text{const}$ are worldlines of constant proper acceleration ξ^{-1}. The surface $\xi = b = \text{const}$ represents the trajectory of the barrier.

The regularized vacuum expectation value $\langle T^\mu_\nu \rangle = \langle 0|T^\mu_\nu|0\rangle$ of the stress-energy tensor far from the conductor ($\xi/b \to \infty$) for a scalar field was found to be

$$\langle T^\mu_\nu \rangle \sim -\frac{1}{2\pi\xi^4}\int_0^\infty \frac{d\omega\,\omega^3}{e^{2\pi\omega}-1}\operatorname{diag}(-1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}) \tag{6a}$$

Thus, $\langle T^{\mu}_{\nu}\rangle$ is reduced below zero by an amount corresponding to blackbody radiation at a temperature

$$T=(2\pi\xi)^{-1} \tag{6b}$$

This asymptotic form is independent of the acceleration of the barrier in the sense that it depends only on the acceleration of the local Killing trajectory.

Equivalently, far from the conductor

$$\begin{aligned}\langle T^{\mu}_{D\nu}\rangle &= [-(480\pi^2\xi^4)^{-1}-(144\xi^4\ln^3\xi/b)^{-1}]\,\mathrm{diag}(-1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3})\\ &\quad +O[(\xi^3\ln^4\xi/b)^{-1}] \qquad (6c)\\ \langle T^{\mu}_{N\nu}\rangle &= [-(480\pi^2\xi^4)^{-1}+(288\xi^4\ln^3\xi/b)^{-1}]\,\mathrm{diag}(-1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3})\\ &\quad +O[(\xi^3\ln^4\xi/b)^{-1}] \qquad (6d)\end{aligned}$$

where $\langle T^{\mu}_{\mathscr{D}\nu}\rangle$ and $\langle T^{\mu}_{N\nu}\rangle$ denote the values of $\langle T^{\mu}_{\nu}\rangle$ for the Dirichlet and Neumann cases, respectively.

The results corresponding to temperature corrections to the Casimir effect (Fiertz, Mehra, Hargreaves, Brown, Maclay, et al.) were generalized by Tadaki and Takagi (1986) for two parallel, infinite plane boundaries in four-dimensional Minkowski spacetime. This system has two special directions (t, z) because of the presence of the boundaries and the heat bath. According to the symmetry of the system, the conservation law, and the tracelessness, $\langle T_{\mu\nu}\rangle$ has the following form:

$$\begin{aligned}\langle T_{\mu\nu}\rangle &= A\,\mathrm{diag}(-1,1,1,-3)+B\,\mathrm{diag}(3,1,1,1)+C\,\mathrm{diag}(1,0,0,1)\\ &\quad +F(z)\,\mathrm{diag}(2,1,1,0) \qquad (7a)\end{aligned}$$

It is remarkable that $\langle T_{33}\rangle\equiv\langle T_{zz}\rangle$ is uniform, though the other diagonal components of $\langle T_{\mu\nu}\rangle$ are possibly z-dependent. The first term in (7) represents the zero-temperature term and the second the Stefan–Boltzmann term. For a conformally coupled massless scalar field

$$\begin{aligned}A&=\frac{\pi^2\hbar c}{1440d^4}, \qquad B=\frac{\pi^2k^4T^4}{90\hbar^3c^3}\\ C&=\frac{\hbar c}{2d^4}\left\{3f(T)-\frac{kd}{\hbar c}Tf'(T)\right\} \qquad (7b)\\ F(z)&=\left[-\frac{1}{2}f(T)+\frac{1}{6}\frac{kd}{\hbar c}Tg'(T,z)\right]\frac{\hbar c}{d^4}\bigg]\end{aligned}$$

where

$$f(T)=-\frac{1}{4\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(2kdT/\hbar c)^4}{[m^2+4n^2(kdT/\hbar c)^2]^2}$$

$$f'(T) = \frac{df(T)}{dT}$$

$$g(T, z) = -\frac{1}{8\pi^2} \sum_{n=-\infty}^{\infty} \sum_{m=1}^{\infty} \frac{(2dkT/\hbar c)^4}{[m^2 + 4(z/d+n)^2(kdT/\hbar c)^2]^2}$$

$$g'(T, z) = \frac{\partial g}{\partial T}$$

In the low-temperature limit ($Td \ll 1$), $\langle T_{\mu\nu} \rangle$ has a sinusoidal z dependence:

$$A = \frac{\pi^2 \hbar c}{1440 d^4}, \qquad B = 0$$

$$C = \frac{\pi k T}{2d^3} e^{-\pi\hbar c/kdT} + O(T^2 e^{-2\pi/T})$$

$$F(z) = \frac{kT}{d^3} e^{-\pi\hbar c/kdT} \left[-\frac{\pi}{6} \cos\frac{2\pi z}{d} + \frac{kdT}{2\hbar c}\left(1 + \frac{kdT}{\hbar c \pi}\right) \times \left(1 - \cos\frac{2\pi z}{d}\right)\right] + O(T^2 e^{-2\pi/T}) \tag{8}$$

Here the Stefan–Boltzmann term is canceled out. The temperature correction is exponentially small, because the basis modes have an energy gap.

In the high-temperature limit ($Td \gg 1$), $\langle T_{\mu\nu} \rangle$ is dominated by the Stefan–Boltzmann value everywhere not close to the boundary. The behavior near the boundary may be seen by considering the single boundary problem. In the limit $d \to \infty$ the result is

$$A = 0, \qquad B = \frac{\pi^2 k^4 T^4}{90 \hbar^3 c^3}, \qquad C = 0$$

$$F(z) = \frac{\pi^2 T^4 k^4}{12 \hbar^3 c^3} (2Z^{-1} \coth Z \operatorname{cosec}^2 Z - Z^{-2} \operatorname{cosec}^2 Z - Z^{-3} \coth Z) \tag{9}$$

where $Z = 2\pi T z k/\hbar c$.

The thermal average deviates from the Stefan–Boltzmann value near the boundary ($Z \ll 1$) due to the T^4 term of $F(z)$:

$$F(z) = -\frac{\pi^2 T^4 k^4}{90 \hbar^3 c^3} \frac{4}{3} \left[1 - \frac{2}{7} Z^2 + \frac{2}{35} Z^4 + O(Z^6)\right] \tag{10}$$

This expansion coincides with the result obtained in Kennedy et al. (1980). The calculations for the electromagnetic field are almost the same.

Finally, Levin et al. (1980) obtained, with the help of the generalized Kirchhoff law,[2] the expressions for the spectral and complete Poynting vector of fluctuating electromagnetic field in a vacuum cavity formed by infinite, flat parallel conductors (ε_1, μ_1) and (ε_2, μ_2) with temperatures T_1 and T_2 ($T_1 > T_2$). The Poynting vector is given by

$$P = \int_0^\infty p(\omega)\, d\omega = \frac{1}{\pi^2} \int_0^\infty (\Pi_1 - \Pi_2) M\, d\omega \tag{11}$$

where

$$\Pi_i = \frac{\hbar\omega}{\exp(\hbar\omega/KT_i) - 1}, \qquad i = 1, 2, \quad k = \frac{\omega}{c}$$

In vacuum ($\varepsilon_1 = \varepsilon_2 = \mu_1 = \mu_2 = h = 1$) for infinite separation ($d \to \infty$) one gets

$$M(\infty) = k^2/8 \tag{12}$$

and under $d = 0$, $M(0) = k^2/4$

$$P(0) = \sigma_{\text{SB}}(T_1^4 - T_2^4) \tag{13}$$

Thus, though each conductor is in equilibrium with radiation, each is so at different temperatures, and the whole system is in the nonequilibrium state. Under these conditions a flow of the fluctuating electromagnetic field from T_1 to T_2 ($T_1 > T_2$) dominates inside the cavity over the flow from T_2 to T_1.

Consider a particle at rest in the gravitational field of a Schwarzschild black hole. Its four-velocity is

$$u^\alpha \equiv dx^\alpha/d\tau = ((1 - 2M/r)^{-1/2}, 0, 0, 0)$$

The proper acceleration of the particle is

$$a^\alpha \equiv \frac{Du^\alpha}{d\tau} = \frac{du^\alpha}{d\tau} + \Gamma^\alpha_{\beta\gamma} u^\beta u^\gamma = \Gamma^\alpha_{tt} u^t u^t$$

($\alpha, \beta, \gamma = t, r, \theta, \varphi$). The only nonvanishing component of Γ^α_{tt} is $\Gamma^r_{tt} = (M/r^2)(1 - 2M/r)$. Hence, $a^\alpha = (0, M/r^2, 0, 0)$,

$$|a| = (g_{\alpha\beta} a^\alpha a^\beta)^{1/2} = (1 - 2M/r)^{-1/2} M/r^2 \tag{14}$$

[2]Levin et al. (1980) point out that the generalized Kirchhoff law contains an expression for the oscillator's average energy $\Theta(\omega, T)$. Nevertheless, zero oscillations have no impact on the energy flow and are discarded here: $\Pi_1 = \Theta(\omega, T) - \hbar\omega/2$. Of course, the energy of the equilibrium fluctuating electromagnetic field is

$$E = \sum_\alpha \hbar\omega_\alpha/2 + \sum_\alpha \hbar\omega_\alpha/\exp(\hbar\omega_\alpha/KT)^{-1}$$

where ω_α are the eigenfrequencies, depending on d.

A stationary distant observer will measure

$$b^{\alpha}=\frac{Du^{\alpha}}{d\tau}\frac{d\tau}{dt}=a^{\alpha}\left(1-\frac{2M}{r}\right)^{1/2}$$
$$|b|\equiv(g_{\alpha\beta}b^{\alpha}b^{\beta})^{1/2}=M/r^{2} \tag{15}$$

Consequently, the peak of the potential barrier (localized in the vicinity of $r=3M$) has a nonzero proper acceleration $\sim(3\sqrt{3}M)^{-1}$.

According to Fabbri (1975), who studied the scattering and absorption of electromagnetic waves by a nonrotating black hole, when the frequency ω of radiation is smaller than the critical frequency ω_c given by

$$\omega_c=(2/27)^{1/2}M^{-1} \tag{16}$$

turning points exist for all partial waves, that is, for all values of l. When $\omega>\omega_c$, turning points exist only for high $-l$ waves; more precisely, they exist if l is greater than the critical parameter l_c given by

$$l_c(l_c+1)=27\omega^2M^2 \tag{17}$$

At high frequencies ($\omega\gg\omega_c$), for $l\ll l_c$, the waves pass above the potential barrier completely unaffected. When l is slightly greater than l_c, the turning points are approximately given by

$$r_{1,2}=3M\left[1\mp\frac{1}{\sqrt{3}}\left(1-\frac{27\omega^2M^2}{l(l+1)}\right)^{1/2}\right] \tag{18}$$

In the case $l<l_c$ the zeros of the wave number are given by

$$\bar{r}_{1,2}=3M\left\{1\mp\frac{i}{\sqrt{3}}\left[1-\frac{l(l+1)}{27\omega^2M^2}\right]^{1/2}\right\} \tag{19}$$

So, for $\omega>\omega_c$ the transmission coefficient of the barrier is

$$T_l=0 \quad \text{at} \quad l>l_c$$
$$T_l=1 \quad \text{at} \quad l<l_c \tag{20}$$

For $\omega\ll\omega_c$ real turning points exist for all partial waves [Eq. (i)]:

$$T_l=4\left[\frac{(l+1)!(l-1)!}{(2l)!(2l+1)!!}\right]^2(2\omega M)^{2l+2} \tag{21}$$

That is why the situation that only low-frequency waves can escape from the region formed by the Casimir plates with reflecting properties is nicely mimicked by the expressions (i). This conclusion is also justified by the calculations of Sanchez (1978): the reflecting properties of the potential barrier provide that Hawking emission is only significant in the frequency range $0 \leq \omega < 1/M$. Consequently, the potential barrier of a nonrotating black hole should be approximated by two concentric shells with the first in the vicinity of the horizon,

$$r = r_1 = 2M + 4\omega^2 M^3 / l(l+1) \to 2M, \qquad \omega \to 0$$

and the second far away from it,

$$r = r_2 \cong l(l+1)^{1/2}/\omega \to \infty, \qquad \omega \to 0$$

The success of the approximation of the Casimir sphere by two parallel plates [equations (1)–(4)] permits us to replace each spherical conductor by two plane conductors.

Consider an observer resting on the surface of one such conductor ($r = r_0$) in the gravitational field of a Schwarzchild black hole. According to the principle of equivalence, this observer is equivalent to an observer accelerated in Minkowski spacetime with proper acceleration $b^{-1} = (1 - 2M/r_0)^{-1/2} M/r_0^2$. However, as is well known (see, for example, Sciama *et al.*, 1981, and the references cited therein), an observer that is accelerated in Minkowski space time with a proper acceleration b^{-1} finds himself in a thermal bath with temperature $T = b^{-1}/2\pi ck$. An observer accelerated with the surface of the wall should find the thermal radiation in equilibrium with the wall at the same temperature. Hence, an observer resting on the surface of a conductor in the gravitational field of a Schwarzschild black hole would discover thermal radiation in equilibrium with a conductor at a temperature

$$T = \frac{M}{r_0^2 (1 - 2M/r_0)^{1/2} 2\pi ck} \tag{22}$$

Consequently, *the interaction of the radiation with the surface of the potential barrier can be described in terms of temperature corrections to the Casimir effect.*

First, the temperature T_1 of a conductor in the vicinity of the horizon is considerably higher than that of a conductor far from it. So, though each conductor is in equilibrium with radiation, the whole system is in the nonequilibrium state ($T_1 > T_2$) and a flow of the fluctuating scalar (or electromagnetic) field establishes itself in the region $[r_1, r_2]$. The flow is directed from the horizon to spatial infinity.

An observer who sits at rest ($r = r_0$) near the horizon will discover a flow of thermal radiation with a temperature

$$T_1 = \frac{1}{2\pi} \frac{M}{r_0^2} \left(1 - \frac{2M}{r_0}\right)^{-1/2}$$

A distant stationary observer at future infinity J^+ will find that the temperature of radiation in the vicinity of the horizon is $T = (1/2\pi) M/r_0^2$.

Indeed, the gravitational blue shift of the photon (ratio of observed energy $\hbar\omega_0$ to energy $\hbar\omega$ emitted at J^+) is

$$\omega_0/\omega=(g_{00})^{-1/2}=(1-2M/r_0)^{-1/2}$$

But $\omega/T=\text{const}$ along the light ray (see Misner et al. 1973). That is why $T_1=T(1-2M/r_0)^{-1/2}$.

According to (15), M/r_0^2 is the magnitude of the acceleration (measured by an observer at J^+) of a particle at rest in the gravitational field of a Schwarzschild black hole. It tends (see Bardeen et al. 1973) to the so-called "surface gravity" $\varkappa$ when the particle is infinitesimally close to the event horizon. For a Schwarzschild black hole, $\varkappa=(4M)^{-1}$ ($c=G=1$). So, the temperature of the radiation near the horizon is $T_1=\varkappa/2\pi$ according to an observer at J^+. Since the temperature T_2 of an observer far from the horizon is negligible, the Poynting vector [see equations (11) and (12)] is

$$\begin{aligned}P&=\int_0^\infty p(\omega)\,d\omega\\&=\frac{1}{\pi^2}\int_0^\infty(\Pi_1-\Pi_2)M\,d\omega\\&=\frac{1}{\pi^2}\int_0^\infty\frac{\hbar\omega M(\infty)\,d\omega}{\exp(\hbar\omega/kT_1)-1}\\&=\frac{1}{\pi^2}\int_0^\infty\frac{\hbar\omega^3\,d\omega}{\exp(\hbar\omega/kT_1)-1}\\&=\frac{\hbar}{\pi^2c^2}\frac{1}{(1-2M/r)^2}\int_0^\infty\frac{\omega_0^3\,d\omega_0}{\exp(\hbar\omega_0 2\pi/k\varkappa)-1}\end{aligned}\tag{23}$$

Equation (23) exactly coincides with the results of the various studies of Hawking radiation made on the basis of usual quantum field theory in curved spacetimes. It should be pointed out that zero oscillations have no direct impact on the energy flow (23). But, of course, they influence it through the expression for the energy of the equilibrium fluctuating electromagnetic field

$$E=\sum_\alpha\{\hbar\omega_\alpha/2+\hbar\omega_\alpha/\exp(\hbar\omega_\alpha/kT)-1\}$$

when the eigenfrequencies ω_α depend on d.

Second, to give a more complete description of the vacuum stress tensor between the conductors and in the whole $[2M,\infty]$ region, we can apply (7a)-(7b) with d the "distance" in the accelerated [or Rindler; see Misner et al. (1973) for details] frame of reference: $d=\xi=(1-2M/r)^{1/2}r^2/M$. But the fact that temperature $T=T(r)$ varies from one point to another hampers the direct utilization of the temperature-correction results. Hence, we shall calculate the $\langle T^{\mu}_{\nu}\rangle_{vac}$ in the vicinities of r_1 and r_2 first. In these regions the variations of T with distance are small in comparison with those in the domain between the conductors.

The proper acceleration of the r_2 barrier is

$$b_2^{-1} = \frac{M}{\Delta_2^2}\left(1 - \frac{2M}{\Delta_2}\right)^{-1/2} \tag{24}$$

where

$$\Delta_2 = \left(\frac{[l(l+1)]^{-1/2}}{\omega} - M\right) \to \infty \qquad \text{if} \quad \omega \to 0$$

A spherical conductor far from the horizon can be represented by two plane conductors with equal temperatures $b_2^{-1}/2\pi$ and accelerations b_2^{-1}. To describe the region $[r_2, \infty]$, the $d \to \infty$ limit of equation (19) should be relevant: $Td \gg 1$, and $\langle T_{\mu\nu}\rangle$ is dominated by the Stefan-Boltzmann value over all the space:

$$A = 0, \qquad C = 0, \qquad B = \frac{\pi^2 T^4}{90} = \frac{M^4}{1440\pi^2 r^8 (1 - 2M/r)^2} \tag{25a}$$

To describe the situation near the other side of the r_2 barrier, it should be noted that the spherical conductor can be exchanged with a pair of flat plates that rest in the Schwarzschild gravitational field. So, taking into account equation (2), we obtain

$$\begin{aligned}\langle T^{\mu\nu}\rangle_{vac} &= \frac{\pi^2}{1440 d^4}\,\text{diag}(-1, 1, 1, 3)\\ &= \frac{\pi^2 M^4}{1440 r^8 (1 - 2M/r)^2}\,\text{diag}(-1, 1, 1, 3)\end{aligned} \tag{25b}$$

Equations (25a)-(25b) are in good qualitative agreement with the exact calculations of Frolov and Zel'nikov (1985) obtained by the usual quantum-field methods for the Boulware vacuum:

$$\begin{aligned}\langle T^{\nu}_{\mu}\rangle = \frac{M^2}{1440\pi^2 r^6}\Bigg[&\frac{(2 - 1.5x)^2}{(1 - x)^2}(-\delta^{\nu}_{\mu} + 4\delta^{\nu}_{0}\delta^{0}_{\mu})\\ &+ 6(3\delta^{\nu}_{0}\delta^{0}_{\mu} + \delta^{\nu}_{1}\delta^{1}_{\mu})\Bigg], \qquad x \equiv \frac{2M}{r}\end{aligned} \tag{26}$$

The Boulware vacuum $|B\rangle$ is defined by requiring that normal modes be of positive frequency with respect to the Killing vector $\partial/\partial t$ with respect to which the exterior region is static. $|B\rangle$ is relevant (Sciama et al. 1981) to the region exterior to a massive body that is only just outside its Schwarzschild radius. *The Boulware vacuum corresponds to the familiar concept of* *empty state at large radii,* but is pathological at the horizon since it diverges in the reference frame of a freely falling observer. In the region near the "nonaccelerated" side of the r_2 conductor, equation (25b) corresponds to the absence from the vacuum of blackbody radiation with temperature $T_2 = b_2^{-1}/2\pi$. This means that if thermal radiation were added, the resulting state would be indistinguishable, near r_2, from the usual Minkowski vacuum. The term, $\langle T^{\mu\nu}\rangle_{\text{vac}}$ in equation (25a) is *positive,* which corresponds to the presence in the $[r_2, \infty]$ region (near the "accelerated" side of the r_2 conductor) of the positive virtual radiation.

When the barrier is made from a real conductor that conducts well only at high frequencies, equations (25) should be modified to include the correction term (Nugayev, 1985)

$$\langle T^{\mu\nu}\rangle_{\text{vac}} = T_1 + T_2 = T_1 + \frac{3\Lambda^4}{2\pi^2}\,\text{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3})$$

$$= T_1 + \frac{3\omega_c^4}{2\pi^2(1-2M/r)^2}\,\text{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}) \tag{27}$$

where $\omega_c = (2/27)^{1/2}(1/M)$ is the cutoff frequency for the absorption of massless waves by a nonrotating black hole. The cutoff-dependent part of (27) at infinity corresponds to that of the ordinary photon gas.

The proper acceleration of the r_1 barrier is

$$b_1^{-1} = \frac{M}{(2M+\Delta_1)^2}\left(1 - \frac{2M}{2M+\Delta_1}\right)^{-1/2}$$

where $\Delta_1 = 8M^3\omega^2/l(l+1)$. If we exchange the r_1 conductor with two plates at a distance apart, we can apply equation (2) to describe the situation in the $[2M, r_1]$ region:

$$\langle T^{\mu\nu}\rangle_{\text{vac}} = \frac{\pi^2 M^4}{1440 r^8(1-2M/r)^2}\,\text{diag}(-1, 1, 1, 3)$$

$$+ \frac{3\omega_c^4}{2\pi^2(1-2M/r)^2}\,\text{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}) \tag{28a}$$

Equivalently, to obtain the first part of equation (28a), we can use the $Td \ll 1$ limit of equation (8) with $d \cong 4M(1-2M/r)^{1/2}$:

$$B = 0, \qquad C = 0, \qquad A = \frac{\pi^2}{1440 d^4} \cong \frac{\pi^2}{1440(4M)^4(1-2M/r)^2}$$

$$\langle T_{00}\rangle \cong -\frac{\pi^6}{90(8\pi M)^4(1-2M/r)^2} \tag{28b}$$

Equations (28a) and (28b) are justified by the application for $[2M, r_1]$ of a black hole of equation (5b) valid near the surface of an ideal spherical conductor:

$$\langle\Omega|:u(r):|\Omega\rangle \cong -\frac{1}{\pi^2 d^4 30\varepsilon^3}$$
$$\cong -\frac{M^4}{30\pi^2 r^8(1-2M/r)^2}\frac{1}{\varepsilon^3}$$

where $\varepsilon = 1-(r/2M)(1-2M/r)^{1/2}$. Again the equations obtained are in good agreement with Frolov's equation (26) for the Boulware vacuum. The $|B\rangle$ vacuum in $[2M, r_1]$ is depressed below zero by an amount corresponding to the absence from the vacuum of blackbody radiation at a temperature $T = 1/8\pi M(1-2M/r)$. It is this pure virtual Casimir negative energy that enables the black hole to contract with nonuniform acceleration. The second part of equation (28a) makes it possible to eliminate the pathology of $|B\rangle$ on the horizon.

To describe the situation near the other side of r_1, in the direction of acceleration, we can again apply the $Td \gg 1$ limit of equation (19) with $d \to \infty$. Again $\langle T_{\mu\nu}\rangle$ is dominated by the Stefan–Boltzmann value over all the conductor:

$$A = 0, \qquad C = 0, \qquad B = +\frac{\pi^2 T^4}{90} = \frac{\pi^2}{90(8\pi M)^4(1-2M/r)^2} \tag{29}$$

The expression (25b) for the "unaccelerated" side of the barrier can be obtained in a way that clearly points out its physical significance. If we exchange the r_2 conductor by two plates at a distance $\Delta_2(1-2M/r)^{1/2}$ apart, the $Td \ll 1$ limit of equation (8) can be used to describe the situation in the vicinity of r_2:

$$\begin{aligned} &B = 0, \qquad C = 0, \qquad A = \frac{\pi^2}{1440 d^4} \cong \frac{\pi^2}{1440(1-2M/\Delta_2)^2\Delta_2^2} \\ &\langle T_{00}\rangle = -\frac{\pi^2}{1440(1-2M/\Delta_2)\Delta_2^2} \end{aligned} \tag{30}$$

Of course the result is too rough, but it helps to reveal an important detail of the radiation picture between the r_1 and r_2 conductors: any observer in the $[r_1, r_2]$ region sees two intersecting flows of blackbody radiation. The dominating flow with $T_1 = 1/8\pi M$ comes from the r_1 conductor; the r_2 conductor results in the second flow, of negative energy. It comes from the surface of r_2 and corresponds to the absence from the vacuum of blackbody radiation with $T_2 = (2\pi b_2)^{-1}$, according to an observer at J^+. An observer at $r = r_0$ in $[r_1, r_2]$ sees

$$T_1 = \frac{1}{8\pi M(1 - 2M/r_0)^{1/2}}$$

$$T_2 = \frac{1}{2\pi\xi} = \frac{M}{2\pi r_0^2(1 - 2M/r_0)^{1/2}}$$

since equations (6a)-(6b) guarantee that the asymptotic forms of $\langle T^{\mu\nu}\rangle_{\text{vac}}$ are independent of the acceleration b_2^{-1} of the barrier in the sense that they depend only on the acceleration ξ^{-1} of the local Killing trajectory. The resulting flow

$$\langle T^{\mu\nu}\rangle = \frac{\pi^2}{90}(T_1^4 - T_2^4) = \frac{\pi^2(1 - x^8)}{90(1-x)^2 \cdot (8\pi M)^4}, \qquad x \equiv \frac{2M}{r} \tag{31}$$

in complete agreement with equation (13) for $d = 0$.

Equation (31) is also in good qualitative agreement with Page's (1982) exact formulas obtained for the Hartle–Hawking vacuum:

$$\langle T^\nu_\mu\rangle_H = \frac{\pi^2}{90(8\pi M)^4}\left\{\frac{[1 - (4 - 3x)^2 x^6](\delta^\nu_\mu - 4\delta^\nu_0\delta^0_\mu)}{(1-x)^2} + 24x^6(3\delta^\nu_0\delta^0_\mu + \delta^\nu_1\delta^1_\mu)\right\} \tag{32}$$

The Hartle–Hawking vacuum $|H\rangle$ is defined by taking incoming modes to be of positive frequency with respect to the null coordinate on the future horizon, and outgoing modes to be of positive frequency with respect to the null coordinate on the past horizon. $|H\rangle$ corresponds to a black hole in equilibrium with an infinite sea of blackbody radiation. This equilibrium is unstable (see Sciama et al. 1981) since the temperature of the hole varies inversely with its mass. So if, in virtue of a fluctuation, the black hole were to absorb more radiation than it emitted, its mass would increase and hence its temperature would fall. It would absorb more radiation, cool further, etc. On the other hand, if the black hole were initially to emit more radiation than it absorbed, then its temperature would rise. It would radiate more rapidly. So in either case the system as a whole tends to evolve away from equilibrium. However, the stability of the equilibrium can be restored by enclosing the black hole in a suitably small box (Gibbons and Perry, 1978), as we have done already.

Thus, all the thermal radiation is "born" in all the "region" $[r_1, r_2]$ between the conductors. Its blackbody spectrum is due to the interaction of scalar, electromagnetic, etc., fluctuations with the conductor surfaces. The dominating flow is directed from r_1 to r_2 ($T_1 > T_2$). The particles between the conductors are still virtual ones, and they would remain virtual if this was the case for real black holes. Yet it is only the scattering aspect of the Scwarzschild gravitational field that entered our ideal model. The exchange of the nonrotating black hole potential barrier with two ideal conductors is merely an approximation. The real potential barrier of a black hole forms a "bell" that lasts *continuously* from zero magnitude at the horizon up to zero at spatial infinity, passing through the maximum at $r = 3M$. The reflecting properties (i) and (17)-(21) ensure that the barrier behaves as a real, and not an ideal, conductor, which conducts well at low frequencies, but as the frequencies increase, its conductivity diminishes. So, the Hawking radiation is "born" inside the "bell" formed by a potential barrier of a nonrotating black hole in all the region $[2M, \infty]$. Its blackbody spectrum is due to the interaction of zero-rest-mass field fluctuations with the surface of the "bell." The flow is directed from the $[2M, 3M]$ region to $[3M, \infty]$ tail of the potential barrier. The particles between the walls of the bell are virtual ones. But they can become real after passing through the $[3M, \infty]$ tail, appearing to an observer at future infinity J^+ as real ones, created by the accelerated tail of the potential barrier.

The comparison of our results with those obtained by the usual and more laborious methods of quantum-field theory in curved spacetimes gives one confidence that the proposed picture does not differ significantly from the "real" one. For instance, the situation at $r = 3M$ with no flux corresponds to unstable photons that circle around the horizon of a Schwarzchild black hole.

DISCUSSION

The following objections have been raised against the present point of view:

1. "Since we can do the original calculation in curved space by usual methods of quantum-field theory, what is the point of this approximation? It seems that this point of view contributes nothing new to the literature." (Anonymous referee.)

2. "The author's attempt to localize the origin of Hawking quanta in space is seriously misconceived. As is well-known, the emitted particles have a wavelength comparable to the size of the black hole itself, and their point of origin cannot be specified to the accuracy discussed, any more than one can specify from which region within an atom a photon is emitted." (Anonymous referee.)

3. "The use of a mirror by Davies to model an analogy to black hole differs crucially from the authors' use. Davies employed the mirror as a means of modelling in one space dimension, the centre of coordinates in a radially-symmetric three-dimensional system, to focus attention on the

fate of field modes that propagate through the centre of the collapsing star and out again. In the present paper it is the potential barrier *outside* the object which is being present as a "mirror." Indeed, the author even states [Nugayev, 1985, p. 96] 'the mathematics of Hawking's 1985 paper can be reinterpreted as describing the particle creation by a spherical barrier in flat spacetime.' This seems to attribute the Hawking effect to the existence of the radial potential barrier rather than the collapse of the star. *Such a picture clearly conflicts with the existence of the Hawking effect in two spacetime dimensions, where there is no radial potential barrier.*" (Anonymous referee.)

4. The author "places great importance on the potential barrier at $r = 3M$. My impression from the study of the Rindler model is that the potential barrier does not play an *essential* role in the Hawking process, although of course it affects the details." (Private communication).

5. "Second I don't understand at all why you are inclined to represent the *horizon* also by a barrier (conducting or otherwise). This seems to have little physical justification." (Private communication).

I am grateful for the above criticism: it helps to clarify my point of view. Nevertheless, these arguments miss the target, for the following reasons.

1. The point would seem to be a weighty methodological (or philosophy of science) argument. It would be sound if I had merely proposed a *theory* of the evaporation process. But what is really described is not an isolated theory or a conjunction of theories. It is a *series of theories* characterized by a certain continuity which connects its members. This continuity evolves from a genuine scientific research program (SRP) adumbrated at the start. The SRP developed here aims to reduce the Hawking effect to quantum-field effects in flat spacetime. It tries to fit reality by producing a *sequence* of *ideal models* (or simply theories) which describe the evaporation process with increasing precision. Each model is constructed on the basis of a certain "hard core" and with the help of "positive" and "negative" heuristics in accordance with the liberal standards of the methodology of scientific research programs. The "hard core" consists of the basic assumptions, perhaps the basic view of nature and its constituents, and necessary background theories, which are accepted by scientists as unfalsified. A "positive heuristic" defines problems, outlines the construction of a set of auxiliary hypotheses, foresees anomalies, and turns them into examples, all according to a preconceived plan. A "negative heuristic" tells us what paths of research to avoid.

By the methodology of SRP the evolution of a program is judged in comparison with the evolution of its rivals, not by itself. A research program is called "progressive" if it makes predictions that are confirmed by subsequent research and thus leads to the discovery of novel facts. It is called "stagnating" if it makes no such predictions, but is reduced to absorbing material that was discovered with the help of its rivals. The most important "objective" features offered by the methodology of SRP are progressive changes. To obtain them one has to wait. Although the methodology of SRP judges the evolution of a program over a period of time, it does not judge its aspect at a particular time.

> Can there be any objective (as opposed to socio-psychological) reason to reject a programme, that is, to eliminate its hard core and its programme for constructing protective belts? Our answer, in outline, is that such an objective reason is provided by a rival research programme which explains the previous success of its rival and supersedes it by a further display of heuristic power. However, the criterion of "heuristic power" strongly depends on how we construe "factual novelty". Until now we have assumed that it is immediately ascertainable whether a new theory predicts a novel fact or not. *But the novelty of a factual proposition can frequently be seen only after a long period has elapsed.* (Lakatos, 1970, p. 155)

With regard to the general methodological question, consider the following statement:

> *A new research programme which has just entered the competition may start by explaining "old facts" in a novel way but may take a very long time before it is seen to produce "genuinely novel" facts.* For instance, the kinetic theory of heat seemed to lag behind the results of the phenomenological theory for decades before it finally overtook it with the Einstein–Smoluchowski theory of Brownian motion in 1905. After this, what had previously seemed a speculative reinterpretation of old facts (about heat, etc.) turned out to be a discovery of novel facts (about atoms).
>
> *All this suggests that we must not discard a budding research programme simply because it has so far failed to overtake a powerful rival. We should not abandon it if, supposing its rival were not there, it would constitute a progressive problemshift.* And we should certainly regard a newly interpreted fact as a new fact, ignoring the insolent priority claims of amateur fact collectors. As long as a budding research programme can be rationally reconstructed as a progressive problemshift, it should be sheltered for a while from a powerful established rival. (Lakatos, 1970, p. 157)

Now let us apply all this to black holes. The irrefutable hard core of our program consists in the assertion that the effect of black-hole evaporation can be understood with the help of quantum-field effects in Minkowski spacetime. Its "positive heuristic" should consist of a set of auxiliary hypotheses which define the important problems and install a sequence of models that describe the Hawking process with increasing precision. The positive heuristic of our program consists in the following assertion: To understand the process of particle creation by a black hole, we must replace its gravitational field by a real conductor. The conductor form is determined by a particular model.

The first ideal model of the program—a pair of plane conductors at rest near the horizon of nonrotating black hole—helped to demonstrate that the negative flow of the Casimir energy should cause the area of the horizon to shrink at a rate consistent with the energy flux observed at future infinity. Thus, the first model helped to reinterpret the well-known peculiarity of the Hawking effect in a new way. According to the criteria already mentioned, we should certainly regard a newly interpreted fact as a *new fact.*

However, the primary model appeared to be too primitive, since it provided only qualitative agreement with Hawking's result. Hence, the second stage of the program had to be realized. It consisted in the construction of a more sophisticated model capable of demonstrating that the mere existence of a spherical barrier and of the horizon is sufficient to compel the black hole to produce thermal radiation of a temperature that *exactly* coincides with the results of Hawking (Nugayev, 1982). Hence, the next "new fact" was discovered and we can judge the transition from the first ideal model M_1 to the second M_2 as a "progressive" one.

But even the second ideal model M_2 appeared to be too primitive to provide a satisfactory description, since the vacuum stress tensor diverged in the reference frame of a freely falling observer as $r \to 2M$. The pathology of the second model is due to assumption of ideal conductivity, which is obviously not the case for the spherical potential barrier of a black hole. So, a third stage of the program had to be realized and the potential-barrier finite conductivity term taken into account (Nugayev, 1985). M_3 helped to reveal that particles are "created" in the $[3M, \infty]$ region.

However, even the third ideal model M_3 was able to describe the creation domain only, but not the *mechanism* of black hole evaporation. The cause is obvious: M_3 ignores the potential barrier thickness. Earlier, the barrier was approximated by a *thin* shell, but Fabbri demonstrated that there exist two branches of turning points for the barrier. Consequently, for the purpose of investigating the interaction of virtual particles with the surface of the potential barrier, a fourth ideal model M_4 had to be constructed. It had to represent the potential barrier by *two* conducting concentric shells. One of the shells is situated near the horizon, while the other is far way from it. Model M_4 gave a more accurate description of the evaporation mechanism.

Thus, we can claim that *none* of the $M_i \to M_k$ transitions ($i < k$, $i, k = 1, 2, 3, 4$) was *ad hoc.* In general, all the more or less vague charges of "ad hocness" fall under the following categories used in appraising SRP (Zamar, 1973).

> "Ad-hocness in research programmes is defined not as a property of an isolated hypothesis but as a relation between two consecutive theories. A theory is said to be ad hoc$_1$ if it has no novel consequences as compared with its predecessor.

> It is ad hoc_2 if none of its novel predictions have been actually "verified," for one reason or another the experiment in question may not have been carried out, or—much worse—an experiment devised to test a novel prediction may have yielded a negative result. Finally the theory is said to be ad hoc_3 if it is obtained from its predecessor through a modification of the auxiliary hypotheses which does not accord with the spirit of the heuristic of the programme."

Now, none of the $M_i \to M_k$ transitions can be evaluated as ad hoc_1 or ad hoc_3, though all of them can be judged as ad hoc_2. Yet, this is not an obstacle for realizing the program, since *all* the transitions within its rival ("usual" or "ordinary" quantum field theory in curved spacetime) are ad hoc_2 also. Up to M_4 our program development is characterized by reinterpretation of already known facts. And, as noted earlier, "it may take a very long time before it is seen to produce 'genuinely novel facts'."

2. The particles are "created" in all the region $[3M, \infty]$. They are "born" within the potential barrier inside the "bell" with the maximum at $r = 3M$. Since the bell's dimensions vary from $d = 0$ at $r = 3M$ to $d = [2M, \infty]$, the wavelengths of the emitted particles vary from $\lambda = 0$ at $r = 3M$ to $\lambda = \infty$. Though the point of origin of the particles with $\lambda \sim M$ cannot be specified exactly, the particles with $\lambda \sim 0$ are "created" in the vicinity of $r = 3M$.

3. It is the joint existence of the potential barrier together with the horizon that enables the hole to evaporate. The role of the barrier is as crucial for the occurrence of radiation as the role of the horizon. And "the existence of the Hawking effect in two spacetime dimensions, where there is no radial potential barrier," is not an argument against the program developed, for the following reasons.

First, the necessity of the potential barrier for Hawking emission is supported by Wald's (1977) five axioms. In the absence of any experimental or observational verifications, Wald's conditions are the only available criteria for deciding whether any given renormalization scheme of the vacuum stress tensor is likely to be correct. Of the five postulates, the last one is one of special importance for us:

Axiom 5. Consider a sequence $\{(g_{\mu\nu})_n\}$ of C^∞ spacetime metrics that agree outside a fixed compact region and are such that the components of $(g_{\mu\nu})_n$ and the derivatives of these components up to fourth order (in a fixed chart) converge uniformly to a C^∞ metric $g_{\mu\nu}$ and its derivatives up to fourth order, respectively. Then we require that for fixed "in" or "out" state $\{\langle T_{\mu\nu}\rangle_n\}$ and its derivative up to third order converge (pointwise) to $\langle T_{\mu\nu}\rangle$ and its derivatives up to third order, respectively.

This axiom is a precise mathematical condition, which expresses the intuitive notion that the stress energy contains no "local curvature term."

This term, depending on second-order or higher derivatives of the metric components, would not vary continuously with the metric in the manner required by the fifth axiom. This was shown by Wald with the help of the following heuristic remarks.

> It is well known that in a fixed chart one can view the vacuum Einstein equation $G_{\mu\nu}=0$ as a second order hyperbolic system of equations for the components of the metric. Consider, now, the Einstein Equation with the classical stress-energy tensor $T_{\mu\nu}$ of some field acting as a source. If we fix initial data for the field on some Cauchy surface, we can veiw the field and hence this classical. $T_{\mu\nu}$ as a (nonlocal) functional of the space time metric. Suppose, however, that this nonlocal functional were to include a "local curvature piece" i.e. a term whose value at a point p depends on derivatives of the metric components high than first order at p, e.g., a fourth order term like $\nabla_\mu \nabla_\nu R$ where R is a scalar curvature. In that case, the character of the dynamical evolution of Einstein's equation with source would be entirely different than that of Einstein's equation in vacuum. In the example just quoted, the evolution would have the character of a fourth-order system $\nabla_\mu \nabla_\nu R = -G_{\mu\nu} +$ (nonlocal part of $T_{\mu\nu}$) rather than that of the second order system. (Wald, 1977, p. 9)

According to the correspondence principle, we want the semiclassical theory ($G_{\mu\nu}=\langle T_{\mu\nu}\rangle_{\text{vac}}$) to reduce to general relativity in the classical limit. However, if the quantum energy stress tensor does not satisfy Axiom 5, this is impossible.

Wald applied his fifth postulate to the so-called "conformal anomaly"[3] and arrived at the following results. In two-dimensional model spacetimes studied by Davies *et al.* and others, one can have a conformally flat spacetime—all two-dimensional spacetimes are conformally flat—which is flat outside a compact region and yet has the property that particles of a conformally invariant field are created. This implies that the quantum stress-energy tensor cannot be conformally invariant, even though the classical stress-energy tensor is conformally invariant. Since the tracelessness of the classical stress energy can be viewed as a consequence of its conformal invariance, it is not surprising that the quantum stress energy should lose its tracelessness as well, as Davies *et al.* found. However, it is not at all clear that similar behavior should occur in four-dimensional spacetimes. It is well known that *two-dimensional manifolds have anomalous conformal properties*, and the above phenomenon may merely be a reflection of this fact. In particular, it is not difficult to show that in four dimensions no particle production can occur for a conformally invariant field in a conformally invariant spacetime that is flat outside a compact region. Due to this argument, Wald arrives at the following rather careful conclusion. "*Thus, the type of particle creation effect which occurs in two dimensions and strongly*

[3] This term refers to the claim that the trace of the quantum stress-energy tensor of a conformally invariant field may be nonzero, though the trace of the classical stress energy vanishes identically.

suggests a conformal anomaly in the trace of the stress energy does not occur in 4-dimensions" (Wald, 1977, p. 10). Hence, until there is a special investigation and comparison of particle creation in four and two dimensions, argument 3 against our standpoint is not valid.

Moreover, the careful study of the two-dimensional particle creation reveals the following fascinating detail. Unruh's (1976) study of the collapse of a spherical shell of matter to a black hole in a two-dimensional model, where "the field equations for massless fields are exactly solvable," contains the following important confession: "In two dimensions, the centrifugal barrier which prevents any particle flow through $r = 0$ is absent. *In order to mimic the effect of such a barrier.* I demand that there be no net radial flux at $r = 0$" (Unruh, 1976, p. 872).

4. The potential barrier *does* play an *essential* role in the Hawking process.

First, it is the occurrence of the potential barrier that demarcates the general relativistic description of the black hole from the Newtonian one (see, for instance, Zel'dovich and Novikov, 1973). But does the effect of particle creation by a black hole occur in Newton's theory of gravity, where the naked horizon is not hidden under the potential barrier?

Second, all the mathematics of Hawking's (1975) classic paper can be interpreted as describing particle creation by a spherical barrier in a flat spacetime with a horizon [see pp. 96–97 of Nugayev (1985) for details]. Hence, it is quite understandable why Hawking's final result does not depend on the details of the collapse.

Third, "indeed, it might be said that Hawking (1975) solved the problem of a black hole by approximating it by a two-dimensional moving mirror" (Davies and Fulling, 1977, p. 34).

5. In our treatment the horizon with its one-sided membrane properties is left unchanged. It is not represented by any barrier (conducting or otherwise). One of the conductors of the M_5 model, for example, is situated in the *vicinity* of the horizon only. Of course, the presence of the horizon should influence the vacuum polarization picture and change the Casimir energy. It cannot change the sign of the energy, as a consideration of the collapse process from the very beginning indicates [see Nugayev and Bashkov (1979) for details]. But of course, the formula

$$T^{00} = -\pi^2 \hbar c^2/720 d^4$$

with $d \cong 3M + 3M = 6M$, which describes the vacuum stress-energy inside the $[0, 3M]$ region, should be exchanged for a better one even in the $[2M, 3M]$ domain. But the question of how the influence of the horizon should be taken into account remains a problem for the present program that should be solved in the future.

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